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Bréguet Range Equation?

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Nomenclature

B	=	fineness D/L
c	=	the specific fuel consumption in kilograms per horsepower-hour
D	=	drag, drift
L	=	lift
P	=	engine power
P_t	=	the power of the engine in horsepower at instant t
R	=	range
S	=	wing area
s	=	distance traveled in time t
t	=	time
V	=	cruising speed
W	=	airplane or fuel weight
W_F	=	the total weight of the fuel, including gasoline and oil, in kilograms at departure
W_{F_t}	=	the weight of the fuel consumed until instant t
W_i	=	the total weight of the airplane at departure
$W_t = W_i - W_{F_t}$	=	the weight of the airplane at instant t
η	=	propeller efficiency

Subscripts

f	=	final conditions
F	=	total fuel
i	=	initial conditions
max	=	maximum
p	=	parasite drag
t	=	conditions at instant t

Introduction

AIRCRAFT design researchers are familiar with the following quoted paragraph:

> The complete aircraft design problem involves the simultaneous optimization of the aerodynamic shape of a configuration, the structure to support the loads, and the propulsion system to provide the sufficient thrust required for flight at the desired airspeeds. Therefore, the objective function to be optimized requires a combination of aerodynamic, propulsion performance and structural weight. Since the Bréguet range equation
>
> $$R = \frac{1}{c}\frac{VL}{D} \ln \frac{W_i}{W_f} \tag{1}$$
>
> combines aerodynamic (VL/D), propulsion (c) in the form of specific fuel consumption and structural (W_i/W_f) figures of merit of the aircraft, it can be used as an objective function in aircraft design.

Not identical, but similar language is often used within the design research documents. Küchemann [1] stated that the Bréguet range is an abstract concept and may be regarded as a figure of merit of the whole aircraft. Therefore it is proper to use the formula that is often ascribed to Bréguet in aircraft design and performance research. A simple search on the AIAA World Wide Web site returns that the terms "Bréguet range," "Bréguet equation," or "Bréguet formula" have been cited within a minimum of 182 research papers, 56 journal papers, and 126 meeting papers from 1964 until February 2005. The statistics show that over one-third of the research papers citing Bréguet's range or endurance equation, 20 journal papers and 57 meeting papers, were published during the last five years, mainly due to an increase in the amount of multidisciplinary design optimization research. However, it is interesting to note that most authors, including the author of this manuscript, use the Bréguet range formula without citing a published paper or a recorded speech of Bréguet, the French aviation pioneer. On the other hand, while mentioning the Bréguet range, some research paper authors attempt to cite a published work only by referring to another author instead of Bréguet personally.

Other research reveals that there are about 27 NACA documents published between 1923 and 1958, where the constant angle of attack–constant velocity cruise range is referred as the Bréguet range. Interestingly, none of these documents cite Bréguet's published work or recorded speech. In addition, 17 out of 18 aerodynamics, aircraft design, and performance textbooks published since 1975, and accessed by this author, never cited a document by Bréguet, although the "Bréguet range" term has been used within them.

This brief discussion shows how the constant angle of attack–constant velocity cruise range is anonymously called "the Bréguet range" without citation of the originator's work, just as it is usually done for Newton's laws or Bernoulli's principle. However, there is a mutual agreement within the science community that the three laws of motion were first introduced by Newton [2] in 1687 [3], and Bernoulli's principle was presented in Bernoulli's work [4], published in 1738 [5]. The questions arise at this point. Is there a printed document published by Bréguet? If there is such a document, then is it the first paper or report introducing the constant angle of attack–constant velocity cruise range calculations? If such a document exists, then why it is not cited by almost all of the aerospace researchers? Are we really aware of how this fundamental equation of the cruise range was introduced? This note attempts to answer these questions and to enlighten the aerospace community about the creation of a fundamental formula.

Searching the Roots

Anderson is the first to conduct a historical research to explore who had first introduced the constant angle of attack–constant

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velocity cruise range calculations. Anderson published results of his research on p. 334 of [6] in 1985. According to Anderson, Coffin seems to be the first to present range and endurance formulas in 1919 [7]. Coffin submitted the first and second parts of the report to NACA on 18 September 1918. The formula for the distance flown by a propeller-driven airplane was introduced by Coffin in the second part of [7] as follows:

$$s = \frac{\eta}{c} \frac{L}{D + D_p} \ \ell n \ \frac{W_i}{W} \tag{2}$$

Although the equation is exactly the same as the constant angle of attack–constant velocity cruise range formula attributed to Bréguet, Coffin gives absolutely no references to anybody [6]. However, to confuse matters, we find a few years later, in Diehl [8], the following statement: "The common formula for range, usually credited to Bréguet, is easily derived." Diehl's report then goes on to use Eq. (2), with no further reference to Bréguet [6]. A 1925 report by Diehl [9] also included the term "Breguet's formula for range," but again without any citation.

In 1945 Von Mises [10] stated that the first to systematically study the problem of range and endurance had been Bréguet, and mentioned that the Bréguet's formulas, already widely adopted at that time, were published in [11]. When presenting some milestones in the evolution of aeronautical technology and in the corresponding educational system developed to train aeronautical engineers, McCormick [12] also referred to Bréguet's paper of 1921 [11], but did not cite the paper itself.

According to Bert [13], the earlier published work on the range of aircraft powered by reciprocating engines is due to Coffin [7] in 1919, and later the same equation originated by Coffin was presented independently by Breguet [14]. Bert also stated that a more complicated range equation was derived by Rateau [15].

Bert's claim introduces another person, Auguste Camille Edmond Rateau (1863–1930), into the list of researchers who studied the cruise range performance of airplanes during the early days of aviation, and also introduces another work of Bréguet regarding the cruise range of airplanes. Actually Bréguet's note [14], cited by Bert, was presented by Rateau on behalf of Bréguet during the 29 October 1923 session of the French Academy of Sciences. Within this note, Bréguet stated "The usual formula of the operating range established by us during the war and discussed later in a remarkable way by Rateau [16], supposing an appreciably horizontal trajectory of the airplane is a reasonable approximation, as we will see it" [14]. Whether Bréguet was using the pronoun "us" in plural or singular form is a question. Diehl, probably, was relying on Bréguet's claim, when he stated "The common formula for range, usually credited to Bréguet, is easily derived [8]."

Rateau's presentation [16] mentioned by Bréguet took place during a February 1920 session of the French Academy of Sciences. When Rateau presented his note [16] he introduced the usual formula of range for propeller airplanes in metric units, as given below:

$$R = 270 \frac{\eta}{c} \frac{L}{D} \ \ell n \ \frac{W_i}{W_f} \tag{3}$$

However, interestingly Rateau never mentioned the name of Bréguet during that presentation, rather he cited Devillers' book [17] as the source for Eq. (3). Devillers wrote the foreword of the book on 18 April 1918 in Paris. The twelfth chapter of the book was about the range of airplanes. However, Devillers while describing the range of airplanes never mentioned the name of Bréguet or findings of Bréguet. Devillers also presented the radius of action calculations for bombing missions along with the range calculations. The present author obtained an original copy of Devillers' book [17], once kept in the confidential files archives of the Unites States Navy. An interesting feature of this copy of the book is that a handwritten inscription of one of the past owner's name, "Walter S. Diehl" on front pastedown of the rebound black cloth binding. However, this requires additional research to learn whether the book was really owned by Diehl and whether the handwriting was really authentic. Devillers' description of the operating ranges of airplanes ([17], pp. 178,179) is translated from French to English and presented in the following section.

The Operating Range [17]

We intend to evaluate the maximum distance R which an airplane can fly under zero wind conditions.

For the altimetry correction of the fuel, the specific fuel consumption is constant and independent of the altitude. The weight of the fuel consumed in dt seconds

$$dW_{F_t} = \frac{cP_t}{3600} dt \tag{4}$$

If B is the fineness of the airplane, then the equation of power at instant t

$$W_t BV = 0.77 \cdot P_t \cdot 75 \tag{5}$$

where 0.77 is efficiency of the propeller. By replacing P_t in Eq. (4) with its value obtained from the last equation, it becomes

$$dW_{F_t} = \frac{cW_t \cdot B \cdot V \cdot dt}{3600 \cdot 0.77 \cdot 75} \tag{6}$$

If the trajectory is slightly ascending, then it is possible to approximate the horizontal component of the trajectory element and write

$$V dt = dR$$

Thus finally

$$dW_{F_t} = \frac{cW_t \cdot B \cdot dR}{208,000} \tag{7}$$

and

$$dR = \frac{208,000}{c \cdot B} \cdot \frac{dW_{F_t}}{W_t} \tag{8}$$

At the instant considered, for a given elementary consumption dW_{F_t} the distance traveled dR will be a maximum when B is minimum. Therefore *the pilot must continuously fly at an angle of minimum drag and preserve this angle during the whole trip.* B is then constant and equal to its minimum value. Let us replace W_t in Eq. (8) by $W_i - W_{F_t}$; then it becomes

$$dR = \frac{208,000}{c \cdot B} \cdot \frac{dW_{F_t}}{W_i - W_{F_t}}$$

Let us integrate by remarking that $W_{F_t} = 0$ at the beginning, and $W_{F_t} = W_F$ at the arrival. We obtain

$$R = \frac{208,000}{c \cdot B} \ \ell n \ \frac{W_i}{W_i - W_F} \tag{9}$$

The Term "Fineness"

Devillers used the term fineness as a measure of the aerodynamic efficiency, which was defining the drag-to-lift ratio. During those early years of aviation, European aviators were using the term fineness rather than lift-to-drag ratio. While discussing the aerodynamic efficiency based on a paper by Breguet, Anderson [19] states

> Breguet's measure of aerodynamic efficiency was the drag-to-lift-ratio. (In the United States, it is conventional to work with the reciprocal of that number, namely, the lift-to-drag ratio. However, in Europe, even today, the drag-to-lift ratio is frequently quoted.) Breguet called the drag-to-lift ratio the fineness of the airplane; the smaller the fineness, the more aerodynamically efficient the aircraft. Moreover, he referred to the equation for the range of an airplane, which shows that the range is directly proportional to the lift-to-drag ratio, or inversely proportional to the fineness.

The previous section from Devillers' book [17] shows that the discussion regarding minimum fineness requirement for maximum

Table 1 Chronology of the early constant angle of attack–constant velocity cruise range estimation studies and events

Date	Event
18 April 1918	Devillers wrote the foreword for [17]
18 Sept. 1918	Coffin submitted Part I and Part II of [7]
1 Oct. 1918	A copy of [17] entered into Naval Department Confidential Files archives.
1 Sept. 1919	Coffin submitted Part III of [7]
Feb. 1920	Rateau associated the formula with Devillers' book.
1921	Bréguet's paper in L'Aérophile, Vol. 29, p. 271.
1922	Bréguet's paper in [11]
29 Oct. 1923	Bréguet's note, presented by Rateau, and stating Bréguet's involvement during the war.

range was initially introduced in 1918. Then in the same year, Coffin [7] stated that the tangent to the polar curve must have a L/D value, which means maximum lift-to-drag ratio for minimum thrust. On the other hand, it is interesting to see that Rateau [16] also used the term $(L/D)_{\max}$ for optimum range condition during his presentation of February 1920. These comments regarding lift-to-drag ratio all preceded Bréguet's published papers.

Conclusions

A chronology of the early constant angle of attack–constant velocity cruise range estimation studies and events is summarized in Table 1, based on the literature available to the public. The only reasonable evidence associating the given cruise range calculation with Bréguet is his own note of 1923 stating "The usual formula of the operating range established us during the war and discussed later in a remarkable way by Rateau [16] supposing an appreciably horizontal trajectory of the airplane is a reasonable approximation, as we will see it" [14]. Rateau never objected to this statement, when presenting Bréguet's note. However, both Rateau and Devillers, who had published cruise range calculations well before 1923, never mentioned Bréguet's work. Even, Rateau associated the formula with Devillers' book. Diehl associated the formula with Bréguet, presumably relying on Bréguet's note of 1923, he excluded all mention of Devillers' book.

Devillers and Coffin were the only two researchers writing the range calculations independently in 1918, and there were no others presenting a printed document about the airplane range before that year. Consequently, relying on the printed information, instead of a claim without hard substance, the usual formula of the constant angle of attack–constant velocity range can be called Devillers–Coffin range equation. However, this would never change Bréguet's place in aviation history due to his other unforgettable and valuable contributions.

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